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BBF-003-1016001

Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

July - 2021

Mathematics : P - 08

(Graph Theory & Complex Analysis - II) (New Course)

Faculty Code : 003
Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions out of ten questions.
(2) Number written in the right indicate marks of the question.

1 Attempt any **five** questions out of ten questions.

(a) Answer the following questions briefly : **4**

(1) Define : Simple graph.

(2) Define : Connected graph.

(3) Find the number of edges in a tree with 7 vertices.

(4) Find the nullity of a connected graph with 4 vertices and 6 edges.

(b) Prove that the number of vertices n in a binary tree is always odd. **2**

(c) State and prove first theorem of graph theory. **3**

(d) State and prove the necessary and sufficient condition for a graph G to be disconnected. **5**

2 (a) Answer the following questions briefly : **4**

(1) Define : Regular graph.

(2) What is the degree of an isolated vertex in a graph ?

(3) Find the total number of edges in a complete graph with 4 vertices.

(4) What is the number of pendant vertices in any Binary tree with n vertices ?

(b) What is the number of vertices in the complete graph K_n, if it has 45 edges ? **2**

(c) Prove that a graph is a tree iff it is minimally connected. **3**

(d) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. **5**

3 (a) Answer the following questions briefly : **4**

(1) Define : Separable graph

(2) Define : Acyclic digraphs

(3) Find the rank of an incidence matrix of a connected graph with 4 vertices.

(4) Find the Regions (faces) of a connected planar graph with 4 vertices and 6 edges.

(b) Define : Independent set of vertices and independence number. **2**

(c) If G is a simple, connected planar graph with f regions, n vertices and e edges ($e > 2$), then prove that $e \leq 3n - 6$. **3**

(d) In usual notation prove that $(W_G, \oplus)$ is an abelian group. **5**

4 (a) Answer the following questions briefly : **4**

(1) Define : Self Dual Graph.

(2) Define : Vertex Coloring

(3) Find the number of edges of a connected planar graph with 6 vertices and 3 faces.

(4) Kuratowski's second graph $K_{3,\,3}$ has _______ edges.

(b) Prove that every tree with two or more vertices is 2-chromatic. **2**

(c) Define path matrix and state its properties. **3**

(d) If G is a graph with n vertices, e edges, f faces and K components then prove that $n - e + f = K + 1$. **5**

5 (a) Answer the following questions briefly : **4**

(1) Define : Mapping.

(2) Define : Mobious Mapping.

(3) Find the fixed points of transformation $W = \dfrac{Z-1}{Z+1}$.

(4) Write the critical points of bilinear transformation $W = \dfrac{az+b}{cz+d}$.

(b) Show that $x + y = 2$ transform into the parabola $u^2 = -8(v-2)$ under the transformation $W = Z^2$. **2**

(c) Obtain a transformation of sector $r \leq 1, 0 \leq \theta \leq \dfrac{\pi}{4}$ under the mapping $W = Z^2$. **3**

(d) Show that the composition of two bilinear transformation is again bilinear transformation. **5**

6 (a) Answer the following questions briefly : 4

(1) Define : Rotation Mapping

(2) Define : Conformal Mapping.

(3) Find the image of $|Z+1|=1$ under the mapping $W=\frac{1}{Z}$.

(4) The points which coincide with their transformation are called ________.

(b) Prove that $W=\frac{az+b}{cz+d}$ is conformal mapping. 2

(c) Find the bilinear transformation which maps $Z_1=\infty, Z_2=i, Z_3=0$ onto $W_1=0, W_2=i$ and $W_3=\infty$. 3

(d) Prove that the transformation $(W+1)^2=\frac{4}{z}$ transform the unit circle of W-plane into parabola of z-plane. 5

7 (a) Answer the following questions briefly : 4

(1) Define : Complex sequence.

(2) Define : Power series.

(3) Find Radius of convergence for the series $\sum_{n=1}^{\infty} n!z^n$.

(4) Write expansion of sin z in Taylor's series for $Z_0=0$.

(b) Find the region of convergence and radius of convergence for series $\sum_{n=1}^{\infty}\frac{Z^n}{3^n-1}$. 2

(c) Expand e^z in increasing power of Z. 3

(d) State and prove Taylor's infinite series for an analytic function. 5

8 (a) Answer the following questions briefly : 4

(1) Define : Complex series.

(2) Define : Absolute Convergence series

(3) Give the formula for finding radius of convergence.

(4) Write expansion of coshz in Maclaurian series.

(b) Prove that $\frac{1}{1-z}=1+z+z^2+z^3+\ldots\ldots$. 2

(c) Expand $\frac{z}{(z-1)(z-3)}$ in Laurent's series for region $0<|z-1|<2$. 3

(d) State and prove necessary and sufficient condition for complex sequence $\{Z_n\}$ to be convergent. 5

9 (a) Answer the following questions briefly : **4**

(1) Define : Singular point

(2) Define : Residue of $f(z)$ at pole Z_0.

(3) Find Res $\left(\frac{e^{2z}}{z(z+1)}, 0\right)$.

(4) Find Singular points of $\frac{\cos \pi z}{(z-1)(z-2)}$.

(b) Find the Residue of $f(z) = \frac{z+2}{(z-1)(z-2)}$ at simple pole. **2**

(c) Derive formula finding Residue at $f(z)$ at simple pole Z_0. **3**

(d) Using Residue theorem prove that **5**

$$\int_0^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{2}$$

10 (a) Answer the following questions briefly : **4**

(1) Define : Simple pole of $f(z)$.

(2) Find Res $\left(\frac{\cos z}{z}, 0\right)$.

(3) Which value of n when $\int_C \frac{dz}{(z-a)^n} = 2\pi i$ where C is the circle $|z-a| = r$.

(4) Which contour is used to integrate $\int_0^{\infty} \frac{dx}{1+x^2}$.

(b) Evaluate $\int_C \frac{2z+3}{z(z-1)} dz, C : |z| = 2$. **2**

(c) If Z_0 is the m^{th} order pole of complex function $f(z)$ **3**

then prove that Res $(f, Z_0) = \frac{\phi^{m-1}(z_0)}{(m-1)!}$ where

$\phi(z) = (z - z_0)^m f(z)$.

(d) Using Residue theorem prove that **5**

$$\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta} = \frac{2\pi}{3}$$